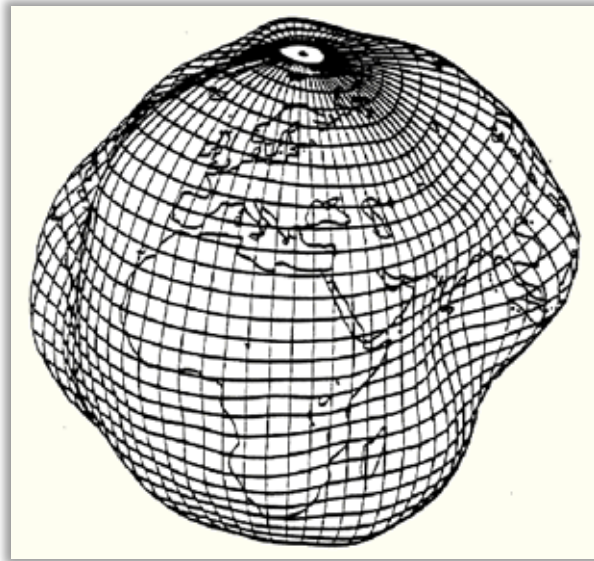


Geodesy

Geodesy is that part of applied mathematics that deals with the size and shape of the earth and the various coordinate systems that are used to represent a position on its surface.

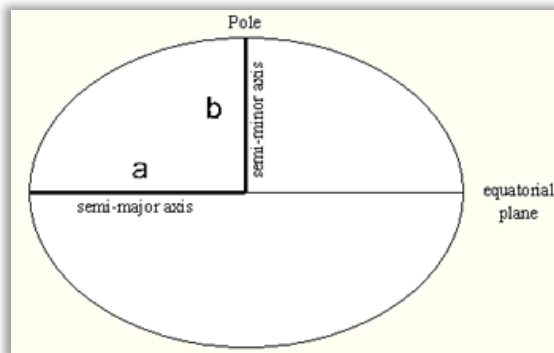
Heights

There are actually three primary surfaces with which we must deal. Those are the actual or topographical surface, the **geoid**, and the **ellipsoid**.



The geoid is a surface of equal gravity potential which closely matches the surface of the sea. It would also be approximated by the water surface that would develop if one could cut into the continents with canals that were connected to the sea. The geoid is undulating and is characterized by regional highs and lows which can vary (relatively) by as much as 100 meters over a distance of 1000 kilometers. Plumb lines and spirit levels are directly affected by the geoid since the gravity vector is perpendicular to its surface. Heights expressed relative to the geoid are termed orthometric heights, and

since the geoid approximates mean sea level, the term **orthometric height** and the term **height above mean sea level** are used interchangeably. The graphic shown here is the geoid surface with a 15,000:1 scaling of the undulations.

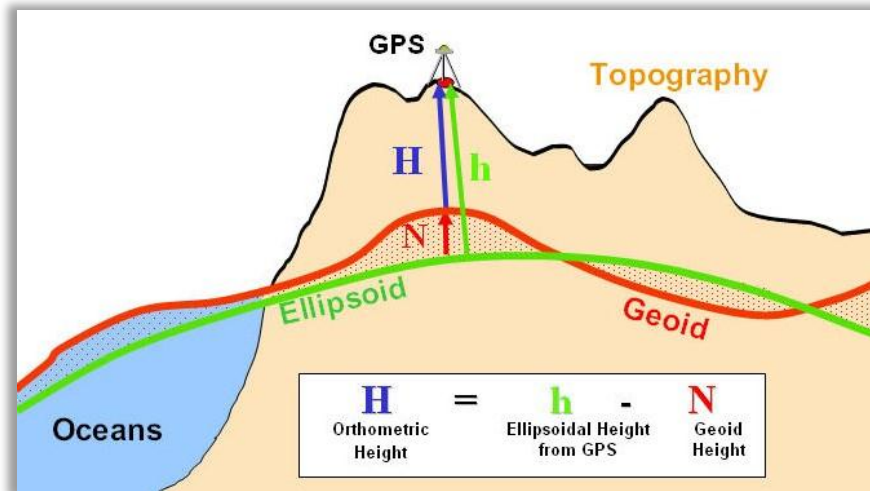


Since any computations on the geoid would be difficult, another less complex surface which adequately describes the earth must be defined. The **ellipsoid** is the surface developed by rotating an ellipse around its minor axis. It is defined mathematically by a semi-major (a in the diagram at left) and semi-minor axis (b). The value resulting from the equation, $a/a-b$ is called the reciprocal of flattening, and is often used with the semi-

major axis to define the ellipsoid. The ellipsoid is a relatively simple mathematical surface on which distance and bearing can be computed and position referenced.

An ellipsoid should be the best representation of the geoid over a desired region. Through astronomical observations and most recently, absolute gravity measurements, numerous sets of ellipsoid constants have been defined for regional and global use.

The ellipsoid and the geoid are two distinctly different surfaces, and because of the undulating nature of the geoid, there are varying differences between the two. These separations are represented by **N** in the figure below.



If ellipsoid heights are represented by **h** and orthometric heights by **H**, the relationship between the two is determined by the equation, $h = H + N$ or conversely $H = h - N$. This is a simple, yet extremely important formula. The equation states that ellipsoid height equals height above mean sea level plus the ellipsoid/geoid separation. A typical value for **N** in the Alberta area is -17 meters, meaning that the geoid is 17 meters below the ellipsoid.

Heights as reported by GPS receivers operating autonomously are ellipsoid heights referenced to the World Geodetic System 1984 (WGS84) ellipsoid. Assuming a GPS surveyor has correctly entered an ellipsoid height for the antenna at his reference station, the same is true for the GPS equipped rovers receiving corrections from that base. Yet the heights as reported in the seismic format coordinate record are orthometric. Therefore, before providing heights, one should convert from WGS84 ellipsoid heights to orthometric heights using the formula, $H = h - N$.

While it seems like an impossible task to determine what that value of **N** might be at a particular location in order to make this conversion, it is not. The same absolute gravity measurements used in determining the most suitable dimensions for a globally used ellipsoid and its relationship to the geoid can be used to provide **N**. In fact, there are several programs which utilize a geoid model in order to compute **N** quite accurately with respect to the WGS84 ellipsoid given an input latitude and longitude. Simply put, these models are used to convert the GPS ellipsoid heights to the orthometric heights we desire.

Datums

While a coordinate system such as latitude and longitude requires a mathematically defined shape on which the parallels and meridians will exist, some defining origin and orientation parameters are also required. The latitude and longitude coordinates on world, continental, and regional maps almost always reference Greenwich as the zero meridian and the poles as 90 degrees north and south. However, accurate geodesy requires far more precise definitions. A **datum** is an adopted ellipsoid and a precise set of defining parameters for a geographic coordinate system.

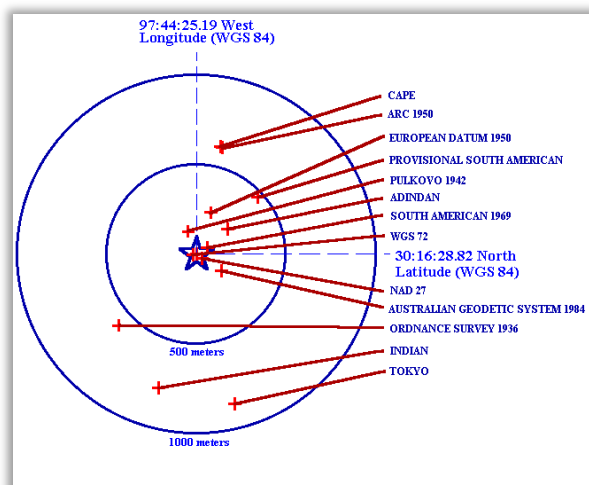
NAD27 Datum:

Lat: 39 13 26.686N
Lon: 98 32 30.506W
a: 6378206.4
b: 6356584
dn: 0
de: 0
az marker: 75 28 09.64
N: 0

Regional datums, sometimes called **local datums**, have as their origin, some physical point and reference azimuth (a second point). For example, the origin of the NAD-27 is a set of reference monuments at Meades Ranch, Kansas. Additional defining parameters include a value of for the geoid/ellipsoid separation at the origin, and the angular orientation of these two surfaces with respect to each other.

World datums, as their name suggests, are utilized worldwide, and their coordinates are determined through various means including protracted observations of satellite data. Since satellites will circle the center of earth's mass, this center could be considered to be the datum's origin. As with regional datums, each world datum has its own adopted ellipsoid.

It is important to point out one particular difference between local and world datums. Local datums are typically inferior in makeup, that is, they inherently contain geodetic warps and tensions that are not present in world datums. This is due to the manner in which the majority of stations were established in these 'older' datums. Despite the incredible care with which traverses were conducted by surveyors with transits and chains, world datums, whose coordinates are established using satellite ranging and other sophisticated techniques, offer far superior relative precision.



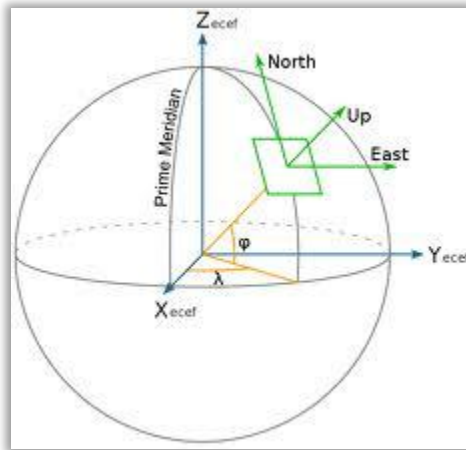
It is impossible to cover the topic of datums with the detail it deserves, but fundamentally, the consequence of having different datums is that a single physical location can be said to have as many sets of geographic coordinates as there are datums for the region. Conversely, the same set of coordinates, but referenced to different datums, have different physical locations on the earth. Therefore, there arises a need to compute coordinates

expressed in one datum given coordinates expressed in another. This is accomplished using one of several well published mathematical transformations called **datum shifts**.
Datum Shifts

The two most common datum shift methods are those based on interpolation, and those that require conversion to and from a three dimensional Cartesian system.

Interpolative datum shifts – Both NADCON and Canada's Nation

Transformation Version 2 (NTv2) are datum shift methods which are interpolative in nature, and are used specifically for converting NAD27 coordinates to and from NAD83 coordinates. It can be interpreted geometrically as a surface of minimum curvature developed from a set of known datum shift values, so that given specific coordinates, a particular set of datum shift values apply.



Cartesian datum shifts - Another datum shift method relies on first converting geographic coordinates to Earth Centered Earth Fixed (**ECEF**) coordinates. The ECEF coordinate system is a 3 dimensional mutually orthogonal Cartesian system which has the center of the mass of the Earth as its origin. The X/Y plane lies along the equator and the Z axis roughly approximates the polar axis.

After coordinates expressed in the first datum are converted to ECEF coordinates, they may undergo either a translation, or a combination of a translation, rotation and scale factor before conversion to the second datum. The former method is called a **three parameter**, or **Molodensky**, datum shift and the latter a **seven parameter**, or **Bursa Wolf** datum shift. The values of both three and seven parameter datum shifts are location dependent when used for coordinate conversion between regional datums, or between world and regional datums, although one set of seven parameter shift values can be used globally between two world datums such as WGS84 and WGS 72.

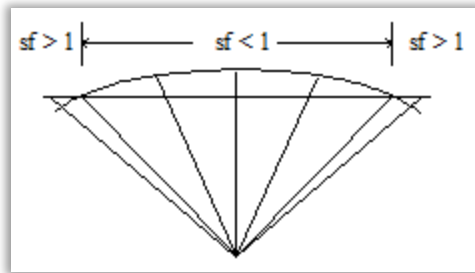
If coordinates in two different datums for the same point are known, three parameter shift values can be determined which are accurate for the immediate area, and perhaps a very large area depending on the similarity of the two datums in question. Seven parameter shift values must be derived by a minimum of three points in the area of interest, and can often be used over a larger area than can the three parameter shift values. However, consideration should be given to the fact that if three or more control points are used, the relative accuracy of the control becomes embedded into the shift. The same holds true for a mean three parameter shift developed by the same points. However, a three parameter shift developed from one station means that any bias in the control point used will be evenly distributed in the immediate area resulting in good

relative accuracy between shifted points.

Also, implicit in the use of an interpolative datum shift method is the assumption that the input regional datum contains inherent distortion, so that a grid of points computed on the datum contains the same relative distortion.

Projections

Projections are geographic to grid conversions carried out through analytic means. There are several types which have very clear geometric interpretations. **Cylindrical** projections involve projecting part of an ellipsoid onto a cylinder, **conic** onto a cone, and **planar** onto a flat plane. Each has certain characteristics which are advantageous for mapping.

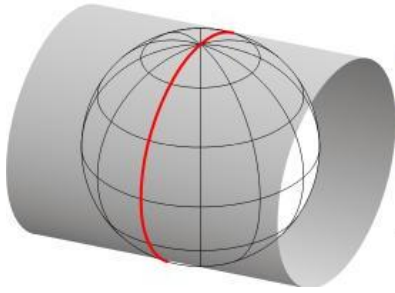


One desirable characteristic of any projection is to maintain a one to one relationship between a distance on the projection surface and the distance it represents on the surface of the ellipsoid. The ratio of these two values is called **scale factor**.
*Grid distance = True distance * SF.*

Geometrically, a one to one scale factor ratio is seen only where the two surfaces touch. In order to maintain a reasonable scale factor over a large area, projection surfaces are made secant to the ellipsoidal surface. Between the secant lines, the scale factor is less than one, and outside, it is greater than one.

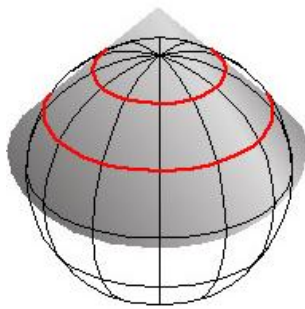
In order to successfully work with maps and coordinates, it is vitally important to know the scale factor with which one is working. In this manner, grid distances can be more accurately converted to real (or near real) distances on the surface of the earth. Equally important is knowledge the **grid convergence** inherent in the conversion. Grid convergence can be viewed as the difference between true north (direction to the geographic north pole) and grid north (direction defined by increasing Y values). Grid convergence increases away from the central meridian. Both scale factor and grid convergence are point dependent and can be computed for each type of projection.

Although a detailed discussion of the different types of projections goes well beyond what can be presented in this appendix, it is instructive to mention the types that are most often used in the survey industry.



A **Transverse Mercator** projection is a cylinder oriented so that secant lines (or a single tangent line) are parallel to a meridian, which means that this projection is a good choice for mapping areas of great north/south extent.

Occasionally, where coastlines or other areas fail to follow the cardinal directions, an **Oblique Mercator** projection is used. This projection relies on a cylinder which is oriented so that secant lines are parallel to the greatest extent of the area to be surveyed or mapped.



A **Lambert Conformal** is a cone oriented so that secant lines form along parallels of latitude which means areas of great east/west extent may be mapped with relatively little scale distortion.

Geodesy As Applied to Seismic

GPS surveying requires that WGS84 control be established for each reference station used during real time operations. Typically, other control points are also established so that the rover can 'tie' or otherwise verify that his real time position closely matches the surveyed position of the control point. Sufficient control should be established in the area of interest for conventional survey backsighting as well. Static and fast static GPS methods using carrier tracking receivers are often used to establish these points.

Static survey baseline processing can be assessed in several ways. This includes evaluating the ratio of the quality of the two best fixed integer solutions, and assessing what is called the reference variance (the topics of least squares and measurements of precision are discussed at length in a separate appendix). Loop closures are also a good means of assessing the precision of the network although some caution is required since factors such as HI measurement error can go undetected.

In adjusting the network of points to be used for land seismic operations, it might be preferable to hold one horizontal station and one vertical station fixed rather than several since the precision with which GPS static surveys can be conducted usually

exceeds the relative precision of the recovered control. Therefore, an adjustment in which two or more second order (i.e., 1:50,000) control points are held fixed has the result of distorting the network of GPS derived stations which would otherwise exhibit several part per million relative accuracy or better. We must realize of course that it is still prudent to deploy receivers on at least two known points (preferably three) so that we may be able to detect a blunder concerning the primary control.

In determining the local datum coordinates for points in the network, the requirement for maintaining the relative precision of the points suggests that the datum shift derived at the station held fixed in the adjustment be used for the entire project. The reasoning is similar to before, that is, if we use an interpolative method, we end up with far less precise coordinates for our stations relative to each other.

Before real time operations can begin, the points to be staked must be uploaded into the data collector. Stakeout involves 'navigating' to geographic coordinates expressed in the WGS84 datum. If stakeout points are provided or initially computed in local datum grid coordinates, they must be transformed to WGS84 geographic coordinates before uploading.

Accurate WGS84 control in the project area may not always be readily available. This means it will occasionally be necessary to derive datum transformation parameters that provide close approximations of WGS84 coordinates. This is acceptable since the differential nature of the real time process provides the relative accuracy we desire, and upon reversing the transformation process, the integrity of the local datum coordinates is preserved.

While it is possible to use local datum coordinates in the reference station and therefore avoid a datum shift altogether, it is not fundamentally correct to do so. Since satellite observations occur within the WGS-84 observation space framework, use of local datum coordinates in a reference station can result in variable rover position errors. These errors are a function of several factors including reference/rover baseline length, satellite geometry, the local datum's relationship to WGS84, and the local datum's internal geodetic inconsistencies.

Height as entered in a reference station should be ellipsoid height. Again, while it is possible to enter orthometric height directly into the receiver in order to avoid post mission conversion of heights from ellipsoidal, it is not advisable to do so for reasons similar to those in the preceding paragraph.

Referring to the schematic for land seismic surveys which appears as the last page of this appendix, we must first obtain or generate the local datum stakeout point coordinates, transform them to geographic coordinates, datum shift them if required, and typically upload a subset of them to the data collector.

Before staking activities, the surveyor must measure the height of the rover antenna and enter the value in the data collector. As the survey is conducted, ellipsoid heights (plus the entered antenna height value) are established since an ellipsoid height is being used at the reference station.

At the end of the survey session, points are downloaded, any required height corrections are made, the resulting ellipsoid heights are converted to orthometric heights, the coordinates are transformed to local grid coordinates, and the resulting coordinates are placed into a desired format.

It should be noted that the same projection and datum shift parameters are used in transformations involving basic control and stakeout positions. Also, reference station ellipsoid height and survey point orthometric heights should be computed using the same geoid model. As with our datum transformation, the geoid model might exhibit some bias, however, it is the relative changes that concern us, not the absolute values. For example, if the OSU91A derived geoid heights were biased by two meters, then the reference station ellipsoidal height would be in error by that amount. However, the geoid height computed at each surveyed point would be applied in the opposite direction, thereby removing the bias. We are counting on OSU91A, in this example, to accurately model the local undulation in computing approximate WGS84 geoid heights.

Internationally, OSU91A, EGM or the DMA model can be used to determine ellipsoid/geoid separations. These are global geopotential models. In the U.S., the GEOID03 model can be used. GEOID03 provides a high resolution model (approximately three second spacing) and a one sigma accuracy of approximately 10cm. In Canada, the NGSD91 and 95 geoidal models, which are similar in resolution to GEOID03, may be utilized.